AdS interpretation of two-point correlation function of QED

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Abstract

We have considered the two-point correlation function of QED in worldline formalism. In position space it has been written in terms of heat kernel. This leads to introducing the $K_1(x_i)$ function, which is related with the bulk-to-boundary propagator of massless scalar field and to reveal bulk-to-boundary propagator in the expression of photon polarization operator.

Introduction

Study of correlation functions is the one of interesting topics of AdS/CFT correspondence. In this connection it has meaning to study the correlation functions of realistic models, such as scalar and spinor QED in the AdS/CFT correspondence framework. Two-point correlation function of electromagnetic field in QED is the photon polarization operator. Worldline formalism on Schwinger parameter moduli space [1,2] turned out useful for rewriting of two and three- point correlation functions of free scalar field theory in terms of heat kernel [3]. Using this approach N-photon amplitudes written in terms of worldline formalism can be rewritten in terms of heat kernel and can be interpreted in the AdS/CFT correspondence language as well. So, we shall use this approach in order to convert the photon polarization operator into the expressions written in terms of bulk to boundary propagators in AdS space-time.

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1 Photon polarization operator

We shall use the expression of photon polarization operator in scalar QED in Schwinger parametrization, which has coinciding expression received both by direct calculation and in worldline formalism [13, 14]:

$$\Pi\left(k_1,k_2\right)=-\frac{\left(g\mu^{\frac{\epsilon}{2}}\right)^2}{\left(4\pi\right)^{2-\frac{\epsilon}{2}}}\int\limits_0^\infty\frac{d\tau}{\tau^{\frac{d}{2}-1}}\int\limits_0^1 d\alpha\ e^{\tau k_1k_2\alpha(1-\alpha)}\left[\frac{2}{\tau}\left(\delta\left(\alpha\right)-1\right)\epsilon_1\cdot\epsilon_2+\left(1-2\alpha\right)^2\left(\epsilon_1\cdot k_2\right)\left(\epsilon_2\cdot k_1\right)\right]\\ \times\int\limits_{-\infty}^{\infty}\frac{d^dz}{\left(2\pi\right)^d}e^{i(k_1+k_2)z}. \tag{1.1}$$

Here d is the dimension of space-time, ϵ_1, ϵ_2 and k_1, k_2 are polarization vectors and momenta of incoming and outgoing photons, τ is the Schwinger parameter. Remark, the expression (1) differs from two-point correlation function of free scalar field theory only by additional square bracket [3]. In position space taking Gaussian integrals over the momenta and derivatives, we obtain the following expression of polarization operator[3]:

$$\Pi\left(x_1,x_2\right)=-C\int\limits_0^\infty\frac{d\tau}{\tau^{\frac{d}{2}-1}}\int\limits_0^1 d\alpha\int\limits_0^1 d\beta\int\limits_{-\infty}^{\infty}d^dz\ \left(\frac{\pi}{\tau\beta\alpha(1-\alpha)}\right)^{\frac{d}{2}}e^{-\frac{(z-x_1)^2}{4\tau\beta\alpha(1-\alpha)}}\left(\frac{\pi}{\tau(1-\beta)\alpha(1-\alpha)}\right)^{\frac{d}{2}}e^{-\frac{(z-x_2)^2}{4\tau(1-\beta)\alpha(1-\alpha)}}\\ \times\left[\frac{2}{\tau}\left(\delta\left(\alpha\right)-1\right)\epsilon_1\cdot\epsilon_2-\left(1-2\alpha\right)^2\frac{1}{2\tau(1-\beta)\alpha(1-\alpha)}\epsilon_1\cdot\left(z-x_2\right)\frac{1}{2\tau\beta\alpha(1-\alpha)}\epsilon_2\cdot\left(z-x_1\right)\right]. \tag{1.2}$$

We can write (2) in terms of the heat kernel $\left(\frac{1}{4\pi t}\right)^{\frac{d}{2}}\ e^{-\frac{(x-z)^2}{4t}}=\left\langle x\left|e^{t\Box}\right|z\right\rangle$ and make change of variables: $\rho_1=\rho\left(1-\beta\right),\ \rho_2=\beta\rho$ and $t=4\tau\rho\beta\left(1-\beta\right)\alpha\left(1-\alpha\right)$:

$$\Pi\left(x_1,x_2\right)=-C'\int\limits_0^\infty\frac{dt}{t^{\frac{d}{2}-1}}\int\limits_0^1 d\alpha\ \ \left[4\alpha\left(1-\alpha\right)\right]^{\frac{d}{2}-2}\int\limits_0^1 d\beta\ \ \left(\rho_1\rho_2\right)^{\frac{d}{2}-2}\rho^{-\left(\frac{d}{2}-2\right)}\int\limits_{-\infty}^{\infty}d^dz\ \left\langle x_1\left|e^{\frac{t}{4\rho_1}\Box}\right|z\right\rangle\\ \times\left\langle x_2\left|e^{\frac{t}{4\rho_2}\Box}\right|z\right\rangle\left[\frac{8\rho_1\rho_2\alpha(1-\alpha)}{t\rho}\left(\delta\left(\alpha\right)-1\right)\epsilon_1\cdot\epsilon_2-\left(1-2\alpha\right)^2\frac{4\rho_1\rho_2}{t^2}\epsilon_1\cdot\left(z-x_2\right)\epsilon_2\cdot\left(z-x_1\right)\right]. \tag{1.3}$$

We can insert into (3) $\Gamma\left(s\right)$ function representations for 1:

$$1=\frac{1}{\Gamma\left(\frac{d}{2}\right)}\int\limits_0^\infty d\rho\rho^{\frac{d}{2}-1}e^{-\rho}. \tag{1.4}$$

Of course, in terms, which contains different degrees of ρ_l, we have to introduce $\Gamma\left(s\right)$ functions having different value of argument. The change of integration variables ρ and β into ρ_1,ρ_2 one's using the equality $\int\limits_0^\infty\rho d\rho\int\limits_0^1 d\beta=\int\limits_0^\infty d\rho_1\int\limits_0^\infty d\rho_2$ is turned out necessary for next step. In this variables $\Pi\left(x_1,x_2\right)$ has got more symmetric form:

$$\Pi\left(x_1,x_2\right)=\int\limits_0^\infty\frac{dt}{t^{\frac{d}{2}+1}}\int\limits_{-\infty}^{\infty}d^dz\int\limits_0^\infty d\rho_1\rho_1^{\frac{d}{2}-1}e^{-\rho_1}\left\langle x_1\left|e^{\frac{t}{4\rho_1}\Box}\right|z\right\rangle\int\limits_0^\infty d\rho_2\rho_2^{\frac{d}{2}-1}e^{-\rho_2}\left\langle x_2\left|e^{\frac{t}{4\rho_2}\Box}\right|z\right\rangle\\ \times\left[-\frac{C_1}{\Gamma\left(\frac{d}{2}+1\right)}t\epsilon_1\cdot\epsilon_2+\frac{C_2}{\Gamma\left(\frac{d}{2}\right)}\epsilon_1\cdot\left(z-x_2\right)\epsilon_2\cdot\left(z-x_1\right)\right]. \tag{1.5}$$

[3]We have included constants into new one

Here we have taken into account $\rho_1 + \rho_2 = \rho$ in the exponent and have included integrals over the α into constants ${C_{1,2}}$[4]:

$$\begin{gathered} C_1 = 4^{\frac{d}{2}-1} C' \int\limits_0^1 d\alpha \ \left[\alpha\left(1-\alpha\right)\right]^{\frac{d}{2}-1}\left(\delta\left(\alpha\right)-1\right), \\ C_2 = 4^{\frac{d}{2}-2} C' \int\limits_0^1 d\alpha \ \left[\alpha\left(1-\alpha\right)\right]^{\frac{d}{2}-1}\left(1-2\alpha\right)^2. \end{gathered}$$

Following [3] we have separated the integrals over the ρ_i and denote them by $K_1\left(x_i, z, t\right)$ function:

$$K_1\left(x_i, z, t\right) = \int\limits_0^\infty d\rho_i \rho_i^{\frac{d}{2}-1} e^{-\rho_i} \left\langle x_i \left| e^{\frac{t}{4\rho_i}\Box} \right| z \right\rangle. \tag{1.6}$$

As was shown in [3], identifying the variable t with the radius of d -dimensional sphere z_0 ($t = z_0^2$) the function $K_1\left(x_i, z, t\right)$ obeys the $d+1$-dimensional Klein-Gordon equation with zero mass:

$$\left[-z_0^2 \partial_{z_0}^2 + \left(d-1\right) z_0 \partial_{z_0} - z_0^2 \Box\right] K_1\left(x, z, t\right) = 0, \tag{1.7}$$

where $\Box$ is the d-dimensional Laplacian in the direction $\overrightarrow{z}$. That means that function $K_1\left(x_i, z, t\right)$ is the bulk to boundary propagator of massless scalar or vector field in the $d+1$-dimensional AdS space-time. Now we can write (5) in terms of this propagator in more suitable form for AdS/CFT interpretation:

$$\begin{gathered} \Pi\left(x_1, x_2\right) = \int\limits_0^\infty \frac{dt}{t^{\frac{d}{2}+1}} \int\limits_{-\infty}^\infty d^d z \ K_1\left(x_1, z, t\right) K_1\left(x_2, z, t\right) \\ \times \ \left[-\frac{C_1}{\Gamma\left(\frac{d}{2}+1\right)} \epsilon_1 \cdot \epsilon_2 t + C_2 \frac{1}{\Gamma\left(\frac{d}{2}\right)} \epsilon_1 \cdot \left(z - x_2\right) \epsilon_2 \cdot \left(z - x_1\right)\right]. \end{gathered} \tag{1.8}$$

Comparing this expression for $\Pi\left(x_1, x_2\right)$ with the two-point correlation function $\Gamma\left(x_1, x_2\right)$ for free scalar field theory, we find additional square bracket factor in our case, which should be replaced by t^3 for last one. Thus, the photon polarization operator is shown in terms of bulk to boundary propagator $K_1\left(x, z, t\right)$ of massless field. For spinor QED case, when we have spinor particles in the loop, the photon polarization operator has form minor changing in its Schwinger parameter expression (1) for scalar loop [1,2]:

$$\begin{gathered} \Pi\left(k_1, k_2\right) = 2 \frac{\left(g\mu^{\frac{\epsilon}{2}}\right)^2}{\left(4\pi\right)^{2-\frac{\epsilon}{2}}} \int\limits_0^\infty \frac{d\tau}{\tau^{\frac{d}{2}-1}} \int\limits_0^1 d\alpha \ e^{\tau k_1 k_2 \alpha\left(1-\alpha\right)} \left[\frac{2}{\tau}\left(\delta\left(\alpha\right)-1\right)\epsilon_1 \cdot \epsilon_2\right. \\ \left. - \left[\left(1-2\alpha\right)^2 - 1\right]\left(\epsilon_1 \cdot k_2\right)\left(\epsilon_2 \cdot k_1\right) + \left(\epsilon_1 \cdot \epsilon_2\right)\left(k_1 \cdot k_2\right)\right] \int\limits_{-\infty}^\infty \frac{d^d z}{\left(2\pi\right)^d} e^{i\left(k_1 + k_2\right) z}. \end{gathered} \tag{1.9}$$

This allows us to remake the formula (7) for spinor loop case (8):

$$\begin{gathered} \Pi\left(x_1, x_2\right) = 2 \int\limits_0^\infty \frac{dt}{t^{\frac{d}{2}+1}} \int\limits_{-\infty}^\infty d^d z \ K_1\left(x_1, z, t\right) K_1\left(x_2, z, t\right) \left[C_1 \frac{\epsilon_1 \cdot \epsilon_2}{\Gamma\left(\frac{d}{2}+1\right)} t\right. \\ \left. + \frac{1}{\Gamma\left(\frac{d}{2}\right)} \ \left[C_2 \epsilon_1 \cdot \left(z - x_2\right) \epsilon_2 \cdot \left(z - x_1\right) - C_3 \epsilon_1 \cdot \epsilon_2 \left(z - x_1\right) \cdot \left(z - x_2\right)\right]\right]. \end{gathered} \tag{1.10}$$

[4] We suppose $d \geq 2$ for convergency of these integrals

Thus, we see from (7) and (9) two-point correlation functions of vector field for scalar and spinor QED are expressed by means of massless bulk-to-boundary propagator. Since here we have studied two-point correlation function of massless vector field (photon field), the obtained result have expressed in terms of bulk to boundary propagator of this field. If we multiply (7) to 2 and add to (9), then we find good agreement with the result obtained in [4]. This tells us that supersymmetry plays important role for matching correlation functions in field theory with the AdS supergravity ones.

References

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